layer u-velocity profiles from the FVSS and FDSS are compared. The results shown in Fig. 1 were obtained on the coarsest grid (33 × 17) with the highest stretching. The result from the FDSS is clearly superior to the FVSS.

Finally, the addition of artificial damping effectively negates the advantage of the FDSS, making its results comparable with the FVSS.

The preceding behavior is explained by detailed examination of the artificial damping terms. Extending the approach of Vatsa et al.[6] the damping term in Eq. (1) is written in the j direction normal to the plate

$$D_{i,j+½} = \frac{1}{2}\tilde{T}\begin{bmatrix} |\tilde{\lambda}_1| & \Delta s \\ |\tilde{\lambda}_2| & \tilde{\rho}\Delta U_\eta \\ |\tilde{\lambda}_3| \tilde{\rho}\Delta J^+/2\tilde{c} & \\ |\tilde{\lambda}_4| \tilde{\rho}\Delta J^-/2\tilde{c} & \end{bmatrix} y_\eta = y_\eta D' \tag{3}$$

where Δs is the difference in entropy between the left and right states, ΔU_η is the jump in normal velocity, and ΔJ^+, ΔJ^- the jumps in the corresponding Riemann invariants at the cell face $i,j + ½$. It has been shown in Ref. 6 that D' is small in a boundary layer, since $\tilde{\lambda}_1$, $\tilde{\lambda}_2 = \tilde{v}$ are small and multiply Δs and $\tilde{\rho}\Delta U_\eta$, which are 0(1); ΔJ^+ and ΔJ^- are also small. The resulting contribution of the damping to the flux difference is

$$\delta D_j = D_{i,j+1/2} - D_{i,j-1/2} = \delta(y_\eta D')_j \tag{4}$$

which is clearly proportional to the grid stretching $y_{\eta\eta}$ in the direction normal to the wall. Since D' is small, the effect of grid stretching on the results will also be small.

In the case of the flux-vector splitting and the central-difference schemes, expressions similar to Eq. (3) can be obtained.[6] Here, however, the damping term D' is not small, since $\tilde{\lambda}_1$ and $\tilde{\lambda}_2$ are not small.[6] The grid stretching in Eq. (4) will, therefore, increase the artificial damping of the scheme and thus contaminate the results in viscous sublayers. The addition of small damping ϵ to $|\tilde{\lambda}_i|$ in the Roe's scheme had the same effect and resulted in errors that were similar to the flux-vector splitting scheme.

Conclusions

The present comparative study of the different upwind schemes for solving the Navier-Stokes equations, as applied to a simple problem, indicates that the flux-difference splitting scheme produces more accurate results on coarse, highly stretched grids than the flux-vector splitting scheme.

Acknowledgment

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Structure of the Contact Discontinuity of Nonstationary Mach Reflections

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THE structure of the contact discontinuity at the triple point of a pseudosteady Mach reflection was reconsidered, as it is known that in some cases the classical "three shock theory" fails to accurately predict the angles between the four discontinuities of the triple point.[1] Based on experimental records, the slipstream was replaced by an angular mixing zone. Integrating this change into the "three shock theory" resulted in very good agreement with the experimental records.

Introduction and Theoretical Background

When a planar shock wave encounters a sharp compressive corner, such as a leading edge of a wedge, in a shock tube, two different types of reflection can occur. They are regular reflection and Mach reflection. The type of reflection which occurs depends for a given gas on the incident shock wave Mach number M_i and the reflecting wedge angle θ_w.

The Mach reflection consists of four discontinuities, the incident shock i, the reflected shock r, the Mach stem m, and the slipstream s. These four discontinuities coincide at the triple point T. Over a plane wedge, the triple point moves along a straight line making an angle X with the reflecting wedge surface. The shock reflection process over plane wedges in shock tubes was found to be self-similar by many experimentalists.[2-5]

By attaching a frame of reference to the triple point, the nonstationary Mach reflection is transformed to a pseudosteady Mach reflection (Fig. 1a). Thus, the shock waves can be treated using the steady flow theory. Assuming that at the vicinity of the triple point the shock waves are straight, the oblique-shock-wave-conservation equations can be applied separately to the shock waves. The conservation equations are

$$\rho_i u_i \sin\phi_j = \rho_j u_j \sin(\phi_j - \theta_j) \tag{1}$$

$$\rho_i \tan\phi_j = \rho_j \tan(\phi_j - \theta_j) \tag{2}$$

$$P_i + \rho_i u_i^2 \sin^2\phi_j = P_j + \rho_j u_j^2 \sin(\phi_j - \theta_j) \tag{3}$$

$$h_i + 1/2 u_i^2 \sin^2\phi_j = h_j + 1/2 u_j^2 \sin^2(\phi_j - \theta_j) \tag{4}$$

where ρ is the density, P the static pressure, h the enthalpy, u the flow velocity, ϕ the incident angle, and θ the deflection angle, $i = 0$ and $j = 1$ for the incident shock, $i = 1$ and $j = 2$ for the reflected shock, and $i = 0$ and $j = 3$ for the Mach stem. If the flow regions are assumed to be in thermodynamic equilibrium, then the above set of 12 equations consists of 18 variables, namely, P_0, P_2, P_3, T_0, T_1, T_2, T_3, u_0, u_1, u_2, u_3, ϕ_1, ϕ_2, ϕ_3, θ_1, θ_2, and θ_3. Usually four of these 18 variables, namely, P_0, T_0, u_0, and ϕ_1 are known as initial conditions. Thus, the above set of 12 equations contains 14 unknowns.

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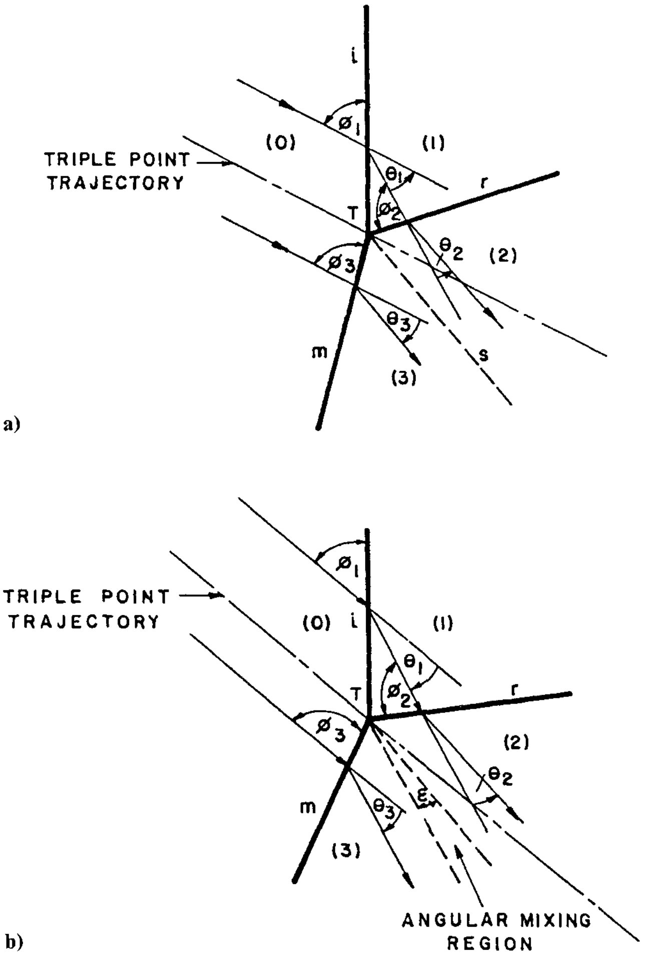

Fig. 1 a) The wave configuration of a Mach reflection from a frame of reference attached to the triple point T, (0)-(3) thermodynamic states, i incident shock wave, r reflected shock wave, m Mach stem, s slipstream, T triple point, ϕ angle of incidence, and θ deflection angle. b) The wave configuration of a steady Mach reflection with an angular mixing region. ϵ is the divergence angle of the angular mixing region.

The two complementary equations needed to make the set solvable arise from the fact that the flow in state (2) behind the reflected shock wave is separated from the flow in state (3) behind the Mach stem [see Fig. (1a)] by a contact discontinuity. Since the pressure on both of its sides must be equal, we have

$$P_2 = P_3 \tag{5}$$

Furthermore, it is common to assume that the contact discontinuity is infinitely thin, i.e., a slipstream. Using this assumption one obtains

$$\theta_3 = \theta_1 - \theta_2 \tag{6}$$

Together with these two complementary equations, we have a set of 14 equations with 14 unknowns, which in principle is solvable. The theoretical model given by Eqs. (1-6) is known as the "three shock theory." A numerical procedure for solving this set of equations is given in Ref. 6.

The assumption that the contact region separating states (2) and (3) is an infinitely thin slipstream was practically adopted by most of the researchers who investigated the Mach reflection phenomenon.[1-10] This assumption was also adopted by all of the well-known textbooks.[11-13]

The shock polar solution of a Mach reflection is also based on this assumption.[6] A typical shock-polar combination of a Mach reflection is shown in Fig. 2. Equations (5) and (6) imply that in the (P, θ) plane, states (2) and (3) coincide at a single point [states (2′) and (3′) in Fig. 2].

In Ref. 1 and in a recent study[14] in which the angles between the various discontinuities were compared to those predicted by the "three shock theory," some serious doubts were raised about the validity of the model described by Eqs. (1-6).

Figure 4 of Ref. 14 illustrates a Mach reflection for $M_i = 2.71$, $\phi_1 = 39.9$ deg, $T_0 = 296$ K, and $P_0 = 760$ Torr. The angles between the various discontinuities as measured from the actual photograph are

$$\omega_{im} = 132 \pm 1 \text{ deg}, \qquad \omega_{ir} = 118 \pm 1 \text{ deg}, \qquad \omega_{rs} = 32 \pm 1 \text{ deg}$$

The inviscid three shock theory given by Eqs. (1-6) results in for a perfect gas assumption

$$\omega_{im} = 132.8 \pm 0.7 \text{ deg}, \qquad \omega_{ir} = 123.0 \pm 1.12 \text{ deg},$$

$$\omega_{rs} = 27.27 \pm 0.58 \text{ deg}$$

The uncertainties of these computed angles are due to the uncertainties in the experimentally measured values of P_0, T_0, M_i and ϕ_1.

It is clearly seen that the predictions based on the classical "three shock theory" fail to accurately predict the angles between the various discontinuities.

If the perfect gas assumption is relaxed and vibrational relaxation is accounted for, then the following results are obtained:

$$\omega_{im} = 131.95 \text{ deg}, \qquad \omega_{ir} = 117.32 \text{ deg}, \qquad \text{and } \omega_{rs} = 29.81 \text{ deg}$$

Although the prediction of ω_{ir} is much improved, the prediction of ω_{rs} is still far from being satisfactory.

Reference 14 presents an attempt to improve the predictions of the "three shock theory" by relaxing the inviscid constraint and accounting for the viscosity via the displacement technique.[13] The predictions of the viscous model for a perfect gas are

$$\omega_{im} = 131.70 \text{ deg}, \qquad \omega_{ir} = 118.02 \text{ deg}, \qquad \omega_{rs} = 32.32 \text{ deg}$$

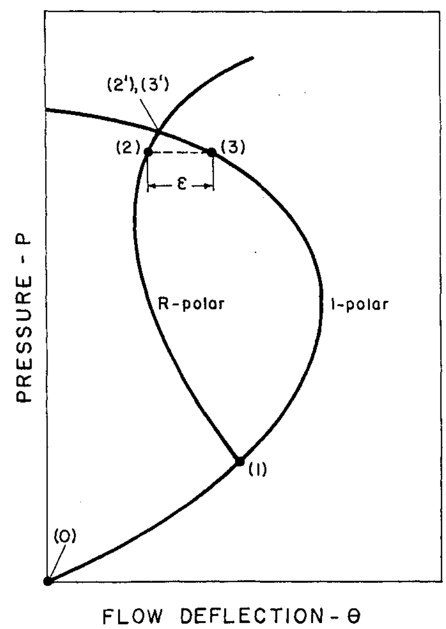

Fig. 2 The shock polar solution of the two Mach reflections shown in Figs. 1a and 1b. States (2′) and (3′) indicate the solution of the Mach reflection shown in Fig. 1a and states (2) and (3) indicate the solution of the Mach reflection shown in Fig. 1b.

These results resemble an excellent agreement with the experimental results for ω_{im}, ω_{ir}, and ω_{rs}. However, due to the fact that it was not clear why an incompressible boundary-layer theory works so well as a perturbation on the inviscid solution, or why displacement thickness ideas work so well in describing what is more probably a free jet flow than a boundary layer, it was decided to try a different approach.

Present Study

The "three shock theory" involves two boundary conditions associated with the contact region separating regions (2) and (3). The first one, which implies that the pressure on both sides of the contact region is constant, is based on physical grounds.[11] The second one is based on the assumption that the contact region is a slipstream. However, an inspection of the Mach reflection under consideration indicates that the so-called contact region diverges as it moves away from the triple point. Therefore, it is possible that the Mach reflection under consideration has an angular mixing region rather than a slipstream. If this indeed is the situation, Eq. (6) is incorrect, and should be replaced by

$$\theta_1 - \theta_2 = \theta_3 - \epsilon \tag{7}$$

where ϵ is the angle of divergence of the angular mixing region.

The possibility for the existence of the above-mentioned three-shock confluence with an angular zone rather than an infinitely thin contact discontinuity was originally introduced by Ref. 11.

The value of ϵ as measured from the actual photograph is 4 ± 0.1 deg. The predictions of the "modified three shock theory" [Eqs. (1-5) and (7)] for a perfect gas are

$$\omega_{im} = 133.95 \text{ deg}, \qquad \omega_{ir} = 123.25 \text{ deg}, \qquad \omega_{rs} = 30.11 \text{ deg}$$

It is obvious that not only did the allowance of an angular mixing region not improve the analytical predictions, but, on the contrary, it made them even worse. However, if the assumption that the gas behaves as a perfect gas is replaced, and the gas is allowed to be in vibrational equilibrium, then the analytical predictions become

$$\omega_{im} = 133.19 \text{ deg}, \qquad \omega_{ir} = 117.38 \text{ deg}, \qquad \omega_{rs} = 31.65 \text{ deg}$$

These values are in very good agreement with those measured experimentally.

Although the predictions given by the viscous model[14] are slightly better than the present ones, it should be mentioned that the present model is much more simple to apply than the one given in Ref. 14.

The foregoing discussion indicates that only the inclusion of real-gas effects was able to improve the analytical predictions of ω_{ir}. This might be due to the fact that ω_{ir} is influenced by the flow behind the reflected shock wave r, which is double shocked and therefore reaches temperatures probably high enough to excite the vibrational degree of freedom. The need to account for vibrational relaxation at moderate shock wave Mach numbers was pointed out by Ref. 6.

It is important to note that the idea of treating the contact discontinuity as an angular mixing region rather than an infinitely thin slipstream in the case of a Mach reflection was introduced originally by Skews[15] who suggested this idea in a departmental report almost two decades ago. Unfortunately, however, he did not support his suggestion quantitatively.

The above results also imply that the shock polar solution of a Mach reflection, such as the one shown in Fig. 2 is incorrect. The shock polar solution of a Mach reflection with $\epsilon \neq 0$ is also shown in Fig. 2. Here states (2) and (3) lie along a constant pressure line, but are separated by the angle of divergence of the angular mixing zone ϵ.

Summary and Conclusions

The present study suggests that although real-gas effects are not important behind moderate incident shock waves, they become significant behind the reflected shock wave where the flow is double shocked.

Furthermore, it was shown that there are cases where the flow behind the reflected shock wave and that behind the Mach stem are separated by an angular mixing region rather than a slipstream. Thus, the boundary condition across this contact region must be modified from Eq. (6) to Eq. (7).

It should be noted, however, that there are probably cases where ϵ is too small, and the use of Eq. (6) does not introduce a meaningful error. Consequently, it should be expected that the angle of divergence of the angular mixing region depends on both the incident shock wave Mach number and the reflecting wedge angle [i.e., $\epsilon = (M_i, \theta_w)$]. For the Mach reflection investigated in this study, $\epsilon(M_i = 2.71,\ \theta_w = 47.1 \text{ deg}) = 4$ deg. The fact that Eq. (6) in the mathematical model of the three shock theory is replaced by Eq. (7) causes the set of 14 equations [i.e., (1-5) and (7)] to become insolvable, for the set of 14 equations consists of 15 unknowns. The additional equation needed to complete the set of equations and make it solvable might be obtained by considering the curvatures of the reflected shock wave and the Mach stem at the triple point.[1] These curvatures force the streamlines behind them to be curved as well, and thereby to diverge. It is possible that the angle of divergence of the angular mixing zone can be correlated to the curvature of the streamlines and their divergence on both sides of the angular mixing zone immediately behind the triple point.

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